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Boiling heat transfer from a vertical row of horizontal tubes

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Abstract

An experimental investigation was carried out to measure the enhancement in the nucleate pool boiling heat transfer of upper heating tubes of copper having 32 mm outer diameter (OD), 18.2 mm internal diameter (ID), and 100 mm effective length, placed one over another in a vertical row as a function of heat flux, type of liquid, and tube material and surface characteristics. Based on the data of present work and similar experiment work of other investigators a model was developed to predict the heat transfer coefficient of individual tube in a multi-tube row and the bundle heat transfer coefficient. The heat flux and pressure range covered was 19–45 kW/m^2 and 35.36–97.5 kPa, respectively. The developed model predicts the experimental data for benzene, toluene, distilled water, and R-113 within ±15%. © 2002 Published by Elsevier Science Ltd.

Keywords: Effectiveness factor

1. Introduction

Boiling heat transfer finds wide application in chemical, petrochemical, food, refrigeration, and other allied industries to generate vapour out of liquid due to its profound ability to transfer enormous amount of heat at low temperature gradients. Boiling heat transfer equipment, in general, are multi-tubular in nature in which process fluid usually remains on the shell side and heating media in the tube side. Historically, the design of heat exchange equipment (reboilers) was based on single tube pool boiling data. The departure of bundle from single tube behaviour was accounted by empirical correction factor. Furthermore, experimental evidence supports a strong dependence on geometry and internal circulation in the shell. Starting with a single tube as a basis, a twin tube arrangement, with one tube above the other, is chosen as the simplest tube bundle to study the heat transfer performance and inter related effects. The results have been used to predict heat transfer coefficient of a tube bundle having a number of heating tubes arranged in a vertical row.

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Boiling heat transfer on twin tube arrangement has been studied earlier in [1–6] but these investigations are not conclusive in nature as effect of heat flux of individual tubes on heat transfer coefficient has not been determined. Therefore, more boiling heat transfer experiments on two or more heating tubes arranged in a vertical row are requited.

The prime objective of the present investigation is to develop a correlation for the prediction of heat transfer coefficient of individual tube as well as tube bundle.

2. Experimental set-up

The experimental rig used in this investigation for the generation of data is shown schematically in Fig. 1. It consists of test vessel (1), heating tubes (2), an electric heater (3), a liquid indicator (4), a condenser (8), a bubbler (10), a vacuum pump (12), and measuring instruments. The test surface is shown schematically in Fig. 2. It is a copper cylinder having the dimensions as shown in the figure. Two tubes of identical dimensions are used. Both of them are fabricated out of the same lot of copper rod. Calibrated copper–constantan thermocouples of 30 gauge (Omega Make) are used to measure

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Nomenclature

c_p	specific heat at constant pressure, J/kg K
c_{sf}	liquid surface combination factor
h	heat transfer coefficient, W/m^2 K
h'_U	heat transfer coefficient of upper tube when both tubes heated simultaneously, W/m^2 K
k	thermal conductivity, W/m K
d	diameter, mm
q	heat flux, W/m^2

Dimensionless numbers

Nu	Nusselt number, hd/k
Nu_B	Nusselt number for boiling, $\frac{h}{k_l}\sqrt{\frac{\sigma}{(\rho_l-\rho_v)}}$
Pr	Prandtl number, $\mu c_p/k_l$
Re_B	Reynold number, $\frac{q}{\mu\lambda}\sqrt{\frac{\sigma}{(\rho_l-\rho_v)}}$

Greek symbols

μ	dynamic absolute viscosity, kg/ms, Ns/m^2
ρ	density, kg/m^3
σ	surface tension, N/m
λ	latent heat of vapourisation, J/kg

Subscripts

b	bulk, boiling, bundle
B	bubble
l	liquid
L	lower heating tube
PL	plain heating tube
PU	plain upper heating tube
U	upper
v	vapour

surface and liquid temperature. The detail of experimental set-up and experimentation is described in [7,8]. Experiments are conducted for two cases – one when tubes are heated separately and another when both the tubes are heated simultaneously with same and different values of heat flux.

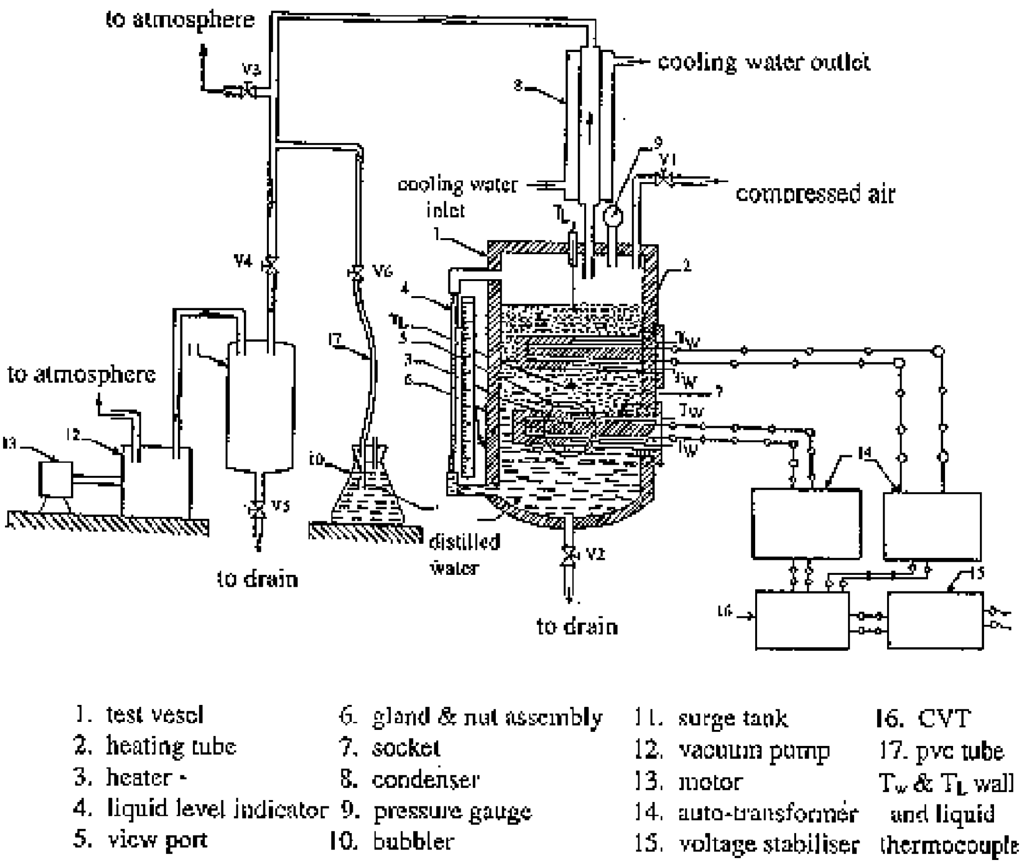

Fig. 1. Schematic diagram of experimental set-up.

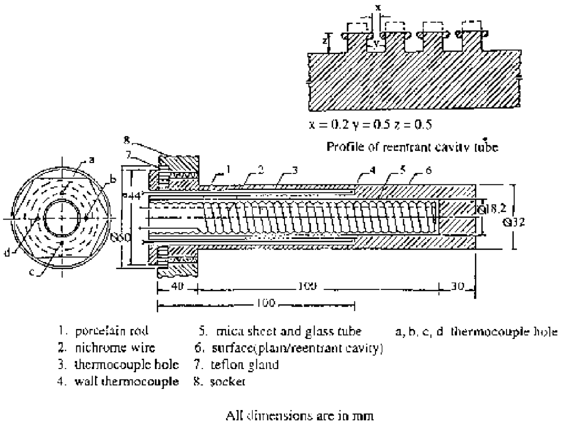

Fig. 2. Details of heating tube tested.

3. Results and discussions

The Experimental data for the boiling of distilled water from two plain tubes arranged in a vertical row are analysed to determine the enhancement of heat transfer due to interaction of lower heating tube with upper heating tube and following functional relationship [7] between average heat transfer coefficient and heat flux for the different cases of heating has been obtained.

When lower heating tube heated alone:

$$h_L = c_1 q_L^{0.7}. \tag{1}$$

When upper heating tube heated alone:

$$h_U = c_2 q_U^{0.7}. \tag{2}$$

When both heating tubes were heated simultaneously at same value of heat flux:

for lower heating tube

$$h_L = c_1 q_L^{0.7}, \tag{3}$$

for upper heating tube

$$h'_U = c_3 q_L^{0.55}. \tag{4}$$

When both heating tubes were heated simultaneously at different values of heat flux:

for lower heating tube

$$h_L = c_1 q_L^{0.7}, \tag{5}$$

for upper heating tube

$$h'_U = c_4 q_L^{0.3}. \tag{6}$$

Initiation of vapour bubble occurs at the preferred sites randomly distributed on the surface of the heating tube. The vapour bubbles grow in size and depart from the heating surface after attaining their maximum size. However, geometry of the heating tube causes hindrance in the free movement of the vapour bubble at some of the circumferential positions. In fact, bubble at the top-most position has free access for its movement, whereas those at the bottom-most position do not have so. Therefore, bubbles at the top-most position have the highest emission frequency, whereas those at the bottom-most position have the least, and those at the side positions have in between the two. Hence, the wall temperature increases continuously from top to side to bottom positions of the heating tube.

Further, above features indicate that simultaneous heating of both the tubes with same value of heat flux as well as different values of heat flux does not affect lower tube but alters the phenomena significantly on upper tube. This is quite obvious as vapour bubbles, after initiation and development, rise upward due to buoyancy force. In this process, lower tube bubbles come in contact with upper tube bubbles and increase the intensity of turbulence there to cause higher heat transfer coefficient. As bubbles from upper tube do not move downward, the turbulence around lower tube remains unaffected and thereby no change in the behaviour of lower tube is observed. Further, the decrease in the value of the exponent in the relationship of h with q on upper tube can be attributed to the turbulence induced by lower tube bubbles on upper tube. This causes convective part of heat transfer to increase and subsequently wall superheat to decrease proportionally. Consequently, vapours bubble population on upper tube decreases. Hence, the value of exponent, and is found to be less than that observed for lower tube.

It is clear from the above relations that heat transfer coefficient of upper heating tube is a function of heat flux of lower tube as well as that of upper tube when both tubes are heated simultaneously. Mathematically, it can be represented as:

$$h'_U = c_5 q_U^m q_L^n \tag{7}$$

or

$$h'_U = c_5 q_U^m q_L^{0.3} \quad \text{from Eq. (6)}. \tag{8}$$

For the case when both the heating tubes are heated simultaneously with same value of heat flux, i.e., $q_L = q_U$, Eq. (8) reduces to

$$h'_U = c_5 q_U^{m+0.3}. \tag{9}$$

From Eqs. (4) and (9), the value of m is found to be 0.25. Hence, Eq. (9) can be rewritten as:

$$h'_U = c_5 q_U^{0.25} q_L^{0.3}, \tag{10}$$

where c_5 is a constant, which depends upon heating surface characteristics and the boiling liquid (distilled water) employed in this investigation. At this junction it

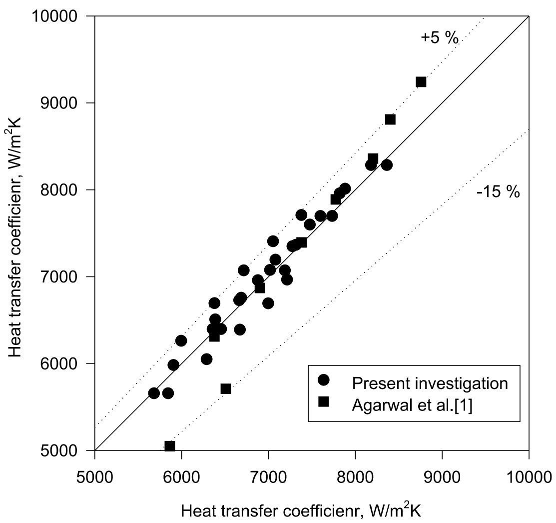

Fig. 3. Comparison of heat transfer coefficient predicted from Eq. (11) with the experimental values for the boiling of distilled water from upper plain heating tube.

may be stated that Eq. (10) reduces to Eq. (4) for the condition of heating both the tubes simultaneously with the same value of heat flux.

The values of constant, c_5 for all the cases when both the tubes were heated simultaneously with same value of heat flux and also for the case when upper heating tube was kept at a fixed value of heat flux and the lower tube heat flux was varied have been determined.

Inserting the value of constant c_5, Eq. (10) assumes the following form:

$$h'_U = 24.463 q_U^{0.25} q_L^{0.3}. \tag{11}$$

Eq. (11) is a simple and convenient equation for the calculation of heat transfer coefficient of upper plain heating tube in a row of two horizontal plain heating tubes when they are heated simultaneously from the knowledge of heat flux of lower and upper heating tubes.

Fig. 3 is a log–log plot between the predicted heat transfer coefficient due to Eq. (11) and the experimental values (heat transfer coefficient) of present investigation and Agarwal [1] for the boiling of distilled water, drawn to examine the validity of Eq. (11). From this plot it can be noted that the predictions match the experimental values well within an error of +5% to −15%, even though these investigations have been conducted on different surface–liquid combinations. Hence, Eq. (11) equation can be used with confidence to determine heat transfer coefficient of upper heating tube.

4. Effectiveness factor of upper heating tube

It is pertinent to evaluate effectiveness of upper heating tube as upper heating tube of the vertical grid of two horizontal tubes offers higher heat transfer coefficient than the lower one due to interaction of latter on the former. Effectiveness factor (EF) is defined as the ratio of heat transfer coefficient of upper tube when heated simultaneously to that when heated individually. Mathematically,

$$\text{EF} = \text{Heat transfer coefficient of upper tube when heated simultaneously}, h'_U/\text{Heat transfer coefficient of upper tube when heated alone}, h_U.$$

From Eqs. (2) and (11), the value of effectiveness factor becomes:

$$\text{EF} = \frac{24.463 q_L^{0.3} q_U^{0.25}}{3.9 q_U^{0.7}} \tag{12}$$

or

$$\text{EF} = 6.27 q_L^{0.3} q_U^{-0.45}.$$

From Eq. (12) it can be stated that effectiveness factor depends upon the heat flux of lower as well as that of upper heating tube – it increases directly with increase in the value of heat flux of lower heating tube but decreases with increase in upper heating tube heat flux.

Fig. 4 shows a comparison between upper tube effectiveness factor as calculated by Eq. (12) and the experimentally obtained values, which includes data due to Agarwal [1] and Bansal [2]. There is an excellent agreement between predictions due to Eq. (12) and experimental values within an error of ±15%. However, data points of each investigation form a distinct group due to differing heating surfaces employed in these investiga-

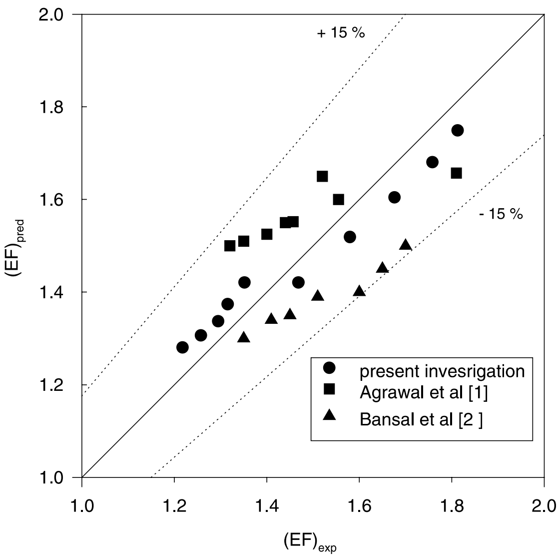

Fig. 4. Comparison of effectiveness factor predicted due to Eq. (12) with experimental values for the boiling of distilled water at atmospheric pressure.

tions. Therefore, Eq. (12) can be used for the determination of upper tube effectiveness factor, EF if the value of constant appearing in it is determined experimentally.

In a vertical row of more than two tubes, interaction of lower tube is bound to occur on upper tubes lying above it and thereby, heat transfer coefficient-heat flux relationship is likely to differ. This naturally will be different for different tube rows and may increase the complexity in design and operation of such a multi-tubular system. Besides, heat transfer coefficient is also likely to be enhanced with the tube row. This complex situation demands generalisation of the data so that these may be employed in the same manner as for single tube.

5. The virtual heat flux of upper tube

As upper heating tube provides higher heat transfer coefficient, it can be thought of as a single tube operating at a virtually enhanced heat flux. This is owing to the fact that higher values of heat transfer coefficients which normally corresponds to enhanced values of heat flux on a single tube are obtained even at low values of heat flux. Therefore, upper heating tube can be regarded to have a virtual heat flux, q'_{U} which is greater than the supplied heat flux, q_{U} and can be expressed as:

$$q'_{\mathrm{U}} = q_{\mathrm{U}} + kq_{\mathrm{L}}, \tag{13}$$

where k represents interaction factor of lower heating tube on the upper one.

Upper heating tube when heated alone behaves in the same manner as a single heating tube. So, h'_{U} can be written as:

$$h'_{\mathrm{U}} = c_6 q'^{0.7}_{\mathrm{U}}$$

or

$$h'_{\mathrm{U}} = c_6 (q_{\mathrm{U}} + kq_{\mathrm{L}})^{0.7}. \tag{14}$$

Reproducing effectiveness factor in terms of heat flux of lower tube and virtual heat flux of upper tube leading to:

$$k = \frac{q_{\mathrm{U}}}{q_{\mathrm{L}}}[(13.77 q_{\mathrm{L}}^{0.428} q_{\mathrm{U}}^{-0.643}) - 1]. \tag{15}$$

For the special case when both the heating tubes are heated simultaneously at the same value of heat flux, i.e., $q_{\mathrm{U}} = q_{\mathrm{L}} = q$, Eq. (15) becomes:

$$k = [13.77 q^{-0.215} - 1]. \tag{16}$$

Thus, by using Eq. (13) along with Eq. (15) or Eq. (16) one can obtain the value of virtually enhanced heat flux, q'_{U} whose application in the single tube relationship of heat transfer coefficient – heat flux will provide heat transfer coefficient of upper tube when both the tubes are heated simultaneously.

It may be emphasised here that the knowledge of the surface–liquid combination factor, c_6 is essential to determine the value of heat transfer coefficient of upper heating tube. Since, its value is likely to vary from surface to surface and its analytical determination is highly improbable due to varying nature of irregularities present on the surface, hence, it can be determined only by boiling heat transfer experiments on a single-tube.

To examine the validity of the concept of fictitious enhanced heat flux of upper heating tube, experimental data due to Agarwal [1] for distilled water, benzene, and toluene on stainless steel surface at atmospheric and sub-atmospheric pressures, Bansal [2] for distilled water, methanol, and isopropanol on brass, copper, and stainless steel surface at atmospheric and sub-atmospheric pressures are used. Since, above equations are dimensional in nature they cannot be employed in the present form to the above experimental data. Therefore, it was thought to use a *Nu–Re–Pr* type correlation, which is very prominently used in boiling heat transfer operation. The use of such a method will eventually involve operating variables and physico-thermal properties of boiling liquids.

Using Eq. (14) following dimensionless equation is developed:

$$Nu_{\mathrm{B}} = c_7 (Re'_{\mathrm{B}})^{0.7}, \tag{17}$$

where Re'_{B} is modified Reynolds number.

Intuitively $(Pr)^{0.4}$ is included in the right-hand side of Eq. (17) so that above correlation may be used to determine heat transfer coefficient of upper heating tube in the assembly of two tubes in a vertical row for all liquids including distilled water at atmospheric and sub-atmospheric pressures. With this modification following correlation is obtained:

$$Nu_{\mathrm{B}} = c_7 (Re'_{\mathrm{B}})^{0.7} (Pr)^{0.4}. \tag{18}$$

Fig. 5 is a plot between the predicted values of Nu_{B} from Eq. (18) and the experimental values of present investigation for the boiling of distilled water at atmospheric pressure. This plot contains the data when upper as well lower heating tube of grid heated with same value of heat flux and also with different values of heat flux. From this plot, it is seen that the predictions match experimental value excellently within a maximum error of ±8%.

Fig. 6 is a plot between $Nu_{\mathrm{B,pred}}$ due to Eq. (18) and the experimentally determined value of $Nu_{\mathrm{B,exp}}$ by Agarwal [1] and Bansal [2] for the boiling of distilled water on two heating tubes arranged in a vertical grid. In both these investigations, upper and lower heating tube have been heated with same value of heat flux. The deviation between the experimental and predicted values is quite natural, as these data have been conducted on heating surfaces of different materials having different

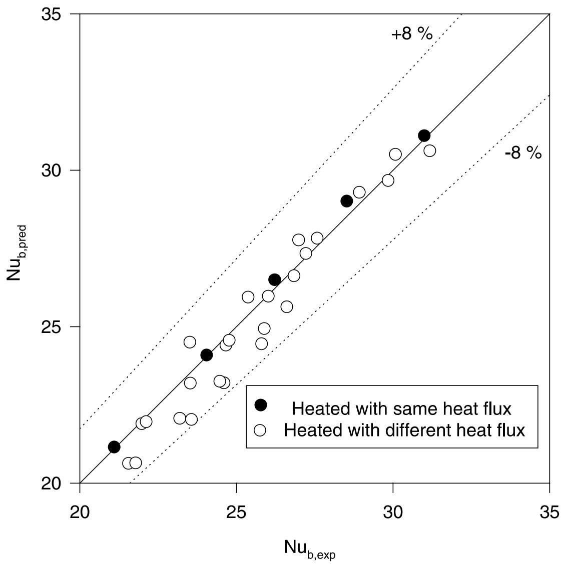

Fig. 5. Comparison of Nusselt number due to Eq. (18) with experimental values of present investigation for the boiling of distilled water at atmospheric pressure.

surface characteristics. Therefore, it was thought to modify the value of constant, c_7 of Eq. (18) so as to fit the experimental data of each investigation. The modified values of constant, c_7 are listed in Table 1. Using the above-modified values of constant in Eq. (18), values of Nu_B were computed for the operating parameters of Agarwal [1] and Bansal [2] and compared with experimentally obtained value of Nu_B due to Agarwal [1] and Bansal [2]. This is shown in Fig. 7.

Table 1
Modified values of constant, c_7 of Eq. (18)

Sl. no	Liquid–surface combination	Cons. c_7
1	Water–copper [present]	69.53
2	Water–copper [2]	83.58
3	Water–brass [2]	81.22
4	Water–stainless steel [2]	73.36
5	Water–stainless steel [1]	70.23
6	Benzene–stainless steel [1]	26.03
7	Toluene–stainless steel [1]	19.49
8	Methanol–stainless steel [2]	42.27
9	Isopropanol–S.S. [2]	36.94
10	Isopropanol–copper [2]	42.69
11	Isopropanol–brass [2]	41.12
12	Methanol–copper [2]	49.16
13	Methanol–brass [2]	47.24

This plot indicates an excellent matching of predictions with the experimental values within an error of ±10%.

Fig. 8 is a plot between $Nu_{B,pred}$ from Eq. (18) and $Nu_{B,exp}$ due to Agarwal [1] and Bansal [2] for the boiling of distilled water on heating tubes of different materials of constructions at atmospheric as well as sub-atmospheric pressures. Predictions are in good agreement with experimental values within an error of +22% to −18%.

Eq. (18) is also tested against liquids other than the distilled water, the experimental data of Agarwal [1] and

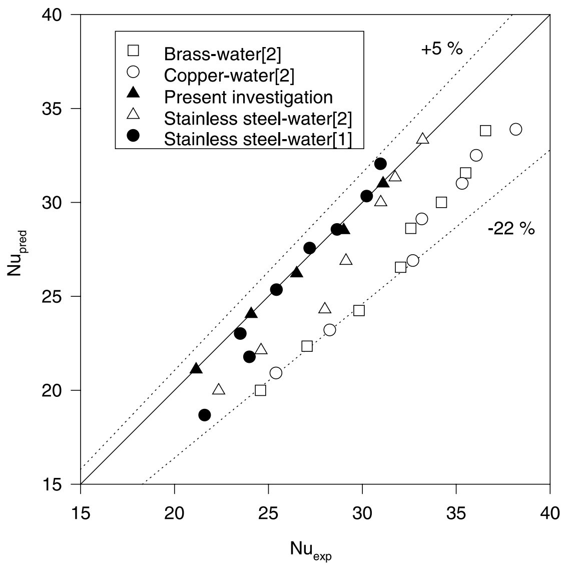

Fig. 6. Comparison of Nusselt number due to Eq. (18) with experimental values for the boiling of distilled water from upper heating tube of various investigators at atmospheric pressure.

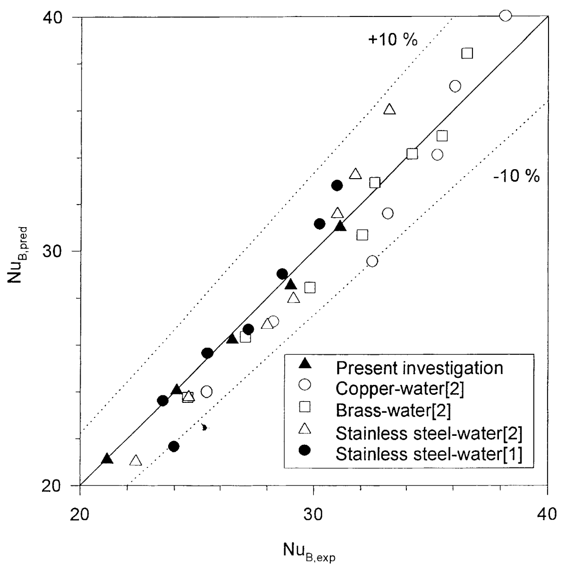

Fig. 7. Comparison of Nusselt number due to Eq. (18) with experimental values for the boiling of distilled water from upper heating tube of various investigators at atmospheric pressure.

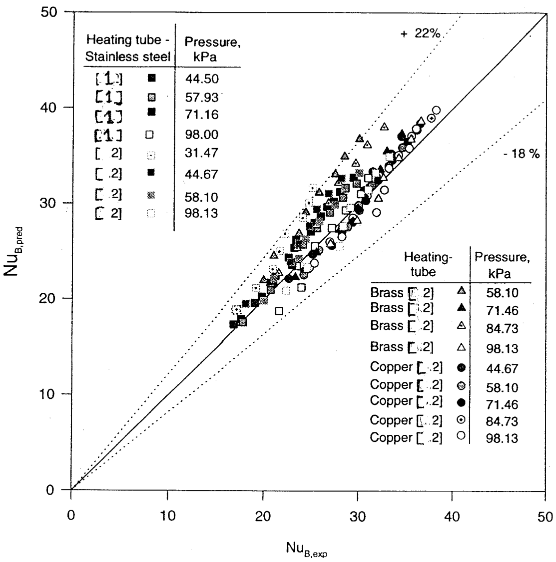

Fig. 8. Comparison of Nusselt number due to Eq. (18) with experimental values for the boiling of distilled water from upper heating tube of various investigators at atmospheric and subatmospheric pressure.

Bansal [2] which include the boiling of benzene, toluene, methanol and isopropanol at atmospheric and subatmospheric pressures have been used, using respective values of constant, c_7 in Eq. (18) the values of Nu_B have been computed for each investigation. The predicted values have been compared with the experimental values in Fig. 9 shows the comparison of these data and are found in excellent agreement within an error band of +10% to −12%.

6. Boiling heat transfer correlation for a grid of horizontal tubes

Reboilers and other heat exchange equipments invariably contain large number of tubes arranged on different pitch arrangements. As a consequence of it, they have vertical grids of horizontal tubes lying one over another. Eq. (18) is applied on such data for its applicability to multi-tubular systems.

Fig. 10 shows a plot between $Nu_{B,pred}$ from Eq. (18) and the experimental values for the second tube-row of the bundle [9]. It is seen from this plot that Eq. (18), under-predicts the experimental values and the maximum error associated with prediction is −15% of experimental values and this deviation is only due to value of constant, c_7.

This principle of fictitious enhanced heat flux can be extended for the analysis of other than twin-tube arrangement. Virtually enhanced heat flux concept of third tube row is defined as given below:

$$q'_{III} = q_{III} + kq'_{II}, \tag{19}$$

where subscripts II and III refer to second and third tube row of the bundle, respectively. However, the value of interaction factor in Eq. (19) has been considered to remain same as defined by Eq. (16). Substituting the value of q'_{II} from Eq. (13):

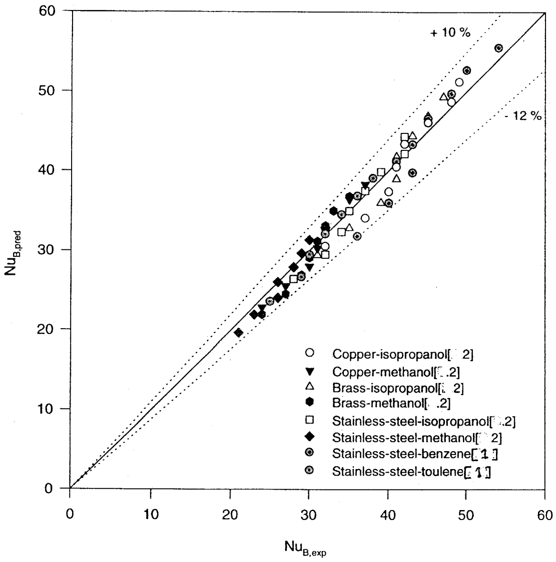

Fig. 9. Comparison of Nusselt number due to Eq. (18) with experimental values for the boiling of different liquids on different surfaces at atmospheric pressure.

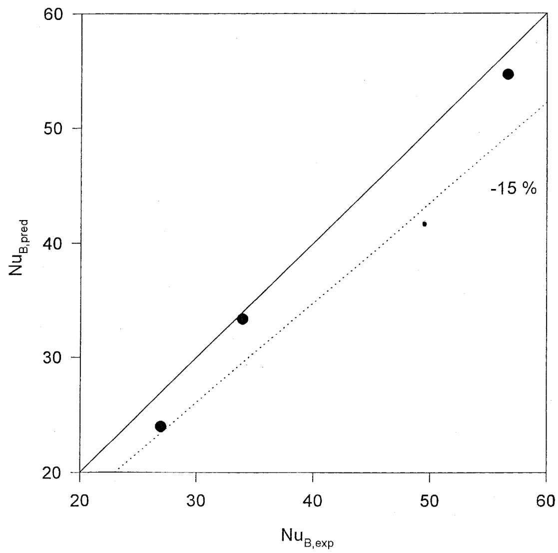

Fig. 10. Comparisons of predictions due to Eq. (18) with experimental values of Leong and Cornwell [9] for the boiling of R-113 on two tubes of a reboiler at atmospheric pressure.

$$q'_{\mathrm{III}} = q_{\mathrm{III}} + k(q_{\mathrm{II}} + kq_{\mathrm{I}}) \tag{20}$$

or

$$q'_{\mathrm{III}} = q_{\mathrm{III}} + kq_{\mathrm{II}} + k^2 q_{\mathrm{I}}.$$

An inspection of Eq. (20) clearly indicates that q'_{III} of third tube is composed of heat flux imposed on third tube, enhancement due to second tube, kq_{II}, and also the enhancement due to the bottom-most tube, $k^2 q_{\mathrm{I}}$. Since, the value of interaction factor, k is smaller than unity, so, the contribution made by the bottom most tube to third tube is smaller than that of second tube. This is quite obvious and natural also as the number of vapour bubbles of bottom-most tube interacting with the third tube is less than that of second tube.

For the special case of heating the tubes with same value of heat flux, Eq. (20) reduces to:

$$q'_{\mathrm{III}} = q(1 + k + k^2). \tag{21}$$

Now, in a reboiler containing n number of tube rows arranged in a vertical grid, virtually enhanced heat flux of nth tube row is expressed by:

$$q'_n = q(1 + k + k^2 + \cdots + k^{n-1})$$

or

$$q'_n = q\left(\frac{1 - k^n}{1 - k}\right), \tag{22}$$

where $k = [13.77q^{-0.215} - 1]$.

Eq. (22) provides virtually enhanced value of heat flux for nth tube row in a vertical grid of n number of horizontal heating tubes from the value of heat flux supplied to it provided all the heating tubes of the grid are heated with same value of heat flux.

Eq. (18) along with Eq. (22) can be used for the determination of heat transfer coefficient of a tube in a given row in a multi-tubular bundle.

7. Conclusions

1. The heat transfer coefficient of upper heating tube of the vertical grid of two horizontal tubes for the boiling of liquid at atmospheric and sub-atmospheric pressures can be evaluated using a simple, convenient, and general equation $Nu_{\mathrm{B}} = c_7(Re'_{\mathrm{B}})^{0.7}(Pr)^{0.4}$, provided liquid surface combination factor is known from single tube experimentation.
2. Using concept of virtual/fictitious enhanced heat flux heat transfer coefficient of a tube in a given row of multi-tubular bundle can be evaluated using equations

$$Nu_{\mathrm{B}} = c_7(Re'_{\mathrm{B}})^{0.7}(Pr)^{0.4} \quad \text{and} \quad q'_n = q\left(\frac{1 - k^n}{1 - k}\right).$$

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